



Believing and Achieving Together

Maths Calculation Guide

September 2018

This calculation policy is organised according to age stage expectations as set out in the National Curriculum 2014, **However it is vital that pupils are taught according to the stage that they are currently working at**, being moved onto the next level as soon as they are ready, or working at a lower stage until they are secure enough to move on.

The National Curriculum 2014 states that the vast majority of children will be able to meet the following objectives, for addition, subtraction, multiplication and division, by the end of each year group.

Year 1

Read, write and interpret mathematical statements involving addition (+), subtraction (−) and equals (=) signs

Represent and use number bonds and related subtraction facts within 20

Add and subtract one-digit and two-digit numbers to 20, including zero

Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = \square - 9$.

Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Year 2

Solve problems with addition and subtraction: using concrete objects and pictorial representations, including those involving numbers, quantities and measures

Applying their increasing knowledge of mental and written methods .

Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:

a two-digit number and ones

a two-digit number and tens

two two-digit numbers

adding three one-digit numbers

Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot

Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers

Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (\times), division (\div) and equals (=) signs

Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot

Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Year 3

Add and subtract numbers mentally, including:

a three-digit number and ones

a three-digit number and tens

a three-digit number and hundreds

Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction

Estimate the answer to a calculation and use inverse operations to check answers

Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables

Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods

Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

Year 4

Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate

Estimate and use inverse operations to check answers to a calculation

Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.

Recall multiplication and division facts for multiplication tables up to 12×12

Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers Recognise and use factor pairs and commutativity in mental calculations

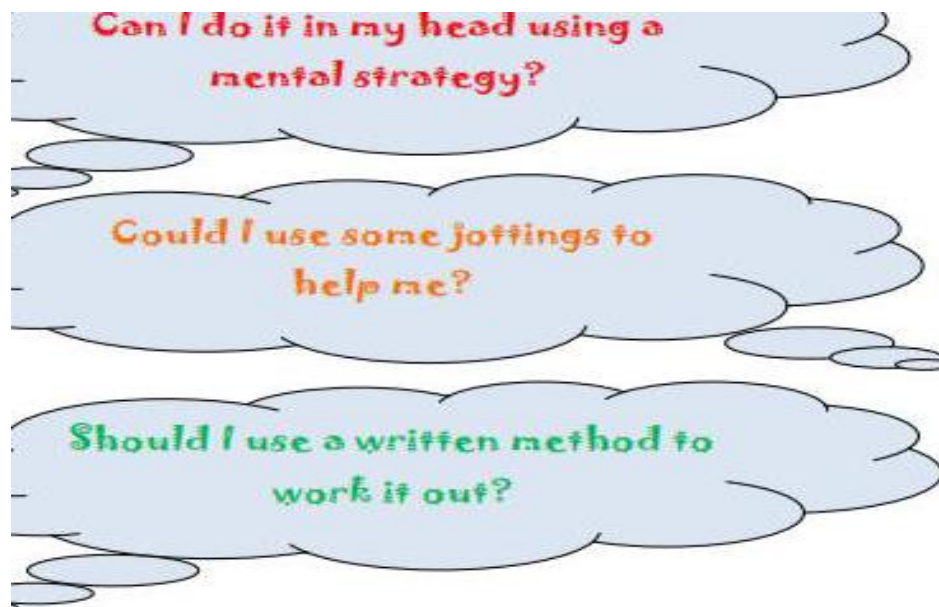
Multiply two-digit and three-digit numbers by a one-digit number using formal written layout

Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

Providing a context for calculation:

It is important that any type of calculation is given a real life context or problem solving approach to help build children's understanding of the purpose of calculation, and to help them recognise when to use certain operations and methods when faced with problems. This must be a priority within calculation lessons.

Choosing a calculation method:



Children need to be taught and encouraged to decide what approach they will take to a calculation, to ensure they select the most appropriate method for the numbers involved.

Addition

Year 1

Add one-digit and two-digit numbers to 20

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line

Children should:

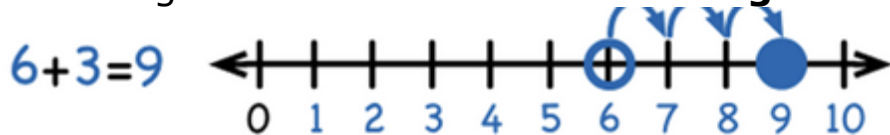
Have access to a wide range of counting equipment, everyday objects, cubes, bead strings, Numicon , Dienes, number tracks, number lines and 100 squares and be shown numbers in different contexts.

Read and write the addition (+) and equals (=) signs within number sentences.

Interpret addition number sentences and solve missing box problems, using concrete objects and number line addition to solve them:

Use numbered number lines to add, by counting on in ones.

Encourage children to start with the **larger** number and count on.



Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.

$$8 + 5$$



Addition

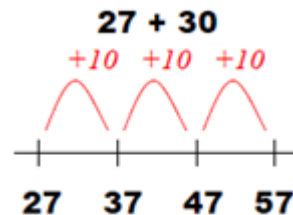
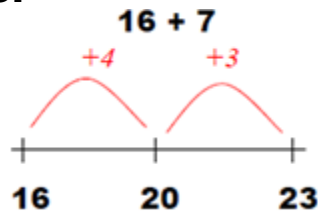
Year 2

Add with two-digit numbers

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary

To develop mental fluency, children should be encouraged to use empty number lines or concrete equipment, e.g. pupils may physically make and carry out the calculation with Dienes apparatus, place value counters, Numicon and a 100 square and then compare their practical version to the written form, to help them to build an understanding of it.

Add 2-digit numbers and units:



Add 2-digit numbers and tens:

Add any 2-digit numbers: $63 + 16 =$
 $60 + 10 = 70$
 $3 + 6 = 9$
 79

	2	3
+	1	7
	1	0
	3	0
	4	0

Add the units first in preparation for the compact method.

In order to carry out this method of addition:

Pupils need to be able to add in columns .

Children need to recognise the value of the hundreds, tens and units without recording the partition.

Addition

Year 3

Add numbers with up to 3-digits

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, compact

	2	3	6
+		7	3
<hr/>			
			9
	1	0	0
	2	0	0
<hr/>			
	3	0	9

Extend the **expanded column addition** method to HTU + TU:

Add the units first in preparation for the compact method.

In order to carry out this method of addition:

Children need to recognise the value of the hundreds, tens and units without recording the partitioning.

Pupils need to be able to add in columns.

Compact column addition:

	2	3	6
+		7	3
	3	0	9
	1		

Move to the compact **column addition** method, with 'carrying'.

Compare the expanded method to the compact column method to develop an understanding of the process and the reduced number of steps involved.

Add the units first

'Carry' numbers underneath the bottom line.

To support understanding, pupils may physically make and carry out the calculation with Dienes apparatus, place value counters, or Numicon and then compare their practical version to the written form, to help them to build an understanding of it. Revisit Year 2 methods, e.g. number lines using 3-digit numbers before progressing to vertical methods.

Addition

Year 4

Add numbers with up to 4-digits

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, compact, thousands, hundreds, digits, inverse

Compact column addition:

Move from expanded addition to the compact column method, **adding units first**, and 'carrying' numbers **underneath** the calculation. Also include money and measures contexts

e.g. $3517 + 396 = 3913$

Add the units first . Carry numbers underneath the bottom line. Reinforce correct place value by reminding them the actual value is 5 hundreds add 3 hundreds, **Not 5 add 3**, for example.

3	5	1	7
	3	9	6
<hr/>			
3	9	1	3
<hr/>			
	1	1	
<hr/>			

Addition

Next steps

Add numbers with more than 4-digits Add several numbers with increasing complexity Add decimals

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, 'carry', expanded, compact, vertical, thousands, hundreds, digits, inverse & decimal places, decimal point, tenths, hundredths, thousandths

Add numbers with more than 4-digits:

e.g. $23481 + 1362 = 2483$

	2	3	4	8	1
+		1	3	6	2
	2	4	8	4	3
			1		

Add decimal numbers:

The decimal point must be aligned in the same way as the other place value columns, and must be in the same column in the answer.

Empty decimal places can be filled with zero to show the place value in each column.

Say '6 tenths add 7 tenths' to reinforce place value.

	1	9	.	0	1	
		3	.	6	5	
+		0	.	7	0	
	<hr/>					
	2	3	.	3	6	
	<hr/>					
	1	1				

Subtraction

Year 1

Subtract from numbers up to 20

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, difference between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_?

Children consolidate understanding of subtraction practically, showing subtraction on bead strings, using cubes, or with Numicon and Dienes apparatus and in familiar contexts. They are introduced to more formal recording using number lines as below:

Count back in ones on a numbered line to take away, with numbers up to 20

$$7 - 3 = 4$$

Subtract by taking away:

0 1 2 3 4 5 6 7

$$17 - 13 = 4$$

0 1 2 3 4 5 6 7 17

-3 -10

Find the difference between:

Introduce in a range of familiar contexts



7

'Seven is 3 more than four'



4

'I am 2 years older than my
sister'

I have 8 sweets and 3 are taken away

Subtraction

Year 2

Subtract with 2-digit numbers

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, difference between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_?, count on, strategy, partition, tens, units

Subtracting pairs of numbers on a number line:

Subtract on a number line by counting back, aim to develop mental subtraction skills. Apparatus which can be used include 100 squares, Dienes and Numicon.

This strategy will be used for: · **2-digit numbers subtract units** (by taking away / counting back) e.g. $36 - 7$ · **2-digit numbers subtract tens** (by taking away / counting back) e.g. $48 - 30$ · **Subtracting pairs of 2-digit numbers** (see below:)

$$47 - 23 = 24$$

Partition the second number

Subtract tens first, then units

$$47 - 10 - 10 - 1 - 1 = 24$$

Move towards more efficient jumps back

$$47 - 20 - 3$$

Subtraction Year 3

Subtracting with 2 or 3-digit numbers

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, difference between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_?, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit

Revisit Year 2 methods, e.g. number lines before progressing to vertical methods. Use Numicom to partition and then subtract. Look at straight forward subtraction before looking at exchange

72 - 47

Introduce exchange through practical subtraction using Dienes or Numicon.

Before subtracting '7' from the 72 blocks, they will need to exchange a row of 10 for ten units.

Then subtract 7, and subtract 4 tens.

$$72 - 47$$



$$\begin{array}{r} 60 \\ \cancel{70} + 2 \\ - 40 + 7 \\ \hline 20 + 5 = \underline{25} \end{array}$$

	2	3	8	-	1	4	6	=	9	2
	¹	⁰	⁰							
	2	0	0	+	1	3	0	+	8	
-	1	0	0	+	4	0	+	6		
			0	+	9	0	+	2		

Expanded column subtraction:

Once pupils are secure with the understanding of 'exchanging', they can use the partitioned column method to support any 2 or 3-digit subtraction.

Use vocabulary 'take'
not 'borrow'.

Subtraction

Year 4

Subtract with up to 4-digit numbers

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, difference between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_?, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit, inverse

Expanded column subtraction:

As introduced in Y3, but moving towards more complex numbers and values.

When subtracting money: partition into £1 + 30 + 5.

2	7	5	4	-	1	5	6	2	=	1	1	9	2
2	0	0	0	+	7	0	0	+	5	0	+	4	
-	1	0	0	+	5	0	0	+	6	0	+	2	
	1	0	0	+	1	0	0	+	9	0	+	2	

Compact column subtraction:

To introduce the compact method, ask children to perform a subtraction calculation with the familiar partitioned column subtraction then display the compact version for the calculation they have done. Ask pupils to consider how it relates to the method they know, what is similar and what is different, to develop an understanding of it.

Children should have opportunities to apply this method to problems involving money and measures.

	2	7	5	4
-	1	5	6	2
	1	1	9	2

Subtraction

Next steps

Subtract with at least 4-digit numbers Subtract several numbers with increasing complexity Subtract decimals

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, difference between, how many more, how many fewer / less than, most, least, count back , how

many left, how much less is_?, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal point, decimal

Compact column subtraction:

Subtract numbers with at least 4-digits

	2	10	1	4	16
-		2	1	2	8
	2	8	9	2	8

Compact column subtraction:

Subtract decimal numbers

The decimal point must be aligned in the same way as the other place value columns, and must be in the same column in the answer.

	6	10	16	8	10
-		3	7	2	• 5
	6	7	9	6	• 5

	0	10	15	3	11	9
-		3	6	• 0	8	0
		6	9	• 3	3	9

Empty decimal places can be filled with **zero** to show the place value in each column.

Multiplication

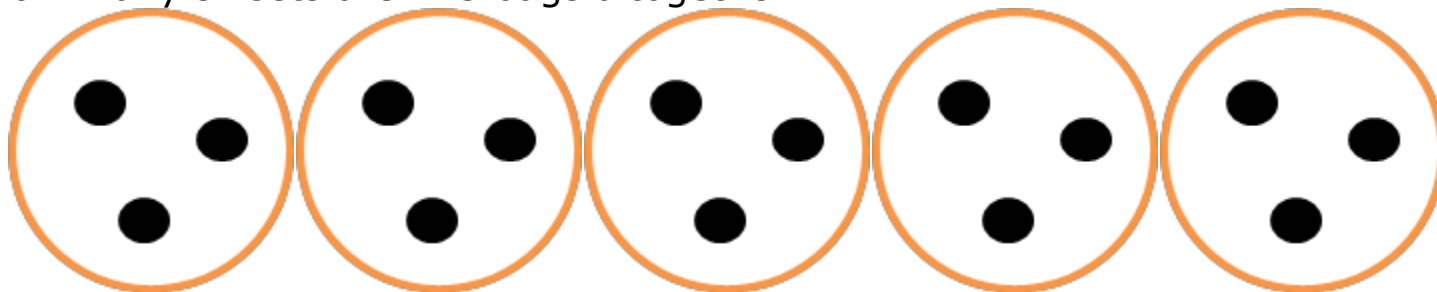
Year 1

Multiply with concrete objects, arrays and pictorial representations

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, once, twice, repeated addition, row, column

There are 3 sweets in one bag.

How many sweets are in 5 bags altogether?



$$3+3+3+3+3 = 15$$

Give children experience of counting equal group of objects in 2s, 5s and 10s.

Present practical problem solving activities involving counting equal sets or groups.

Give children pictorial representations such as arrays.

How many legs will 3 teddies have?



$$2 + 2 + 2 = 6$$

Multiplication

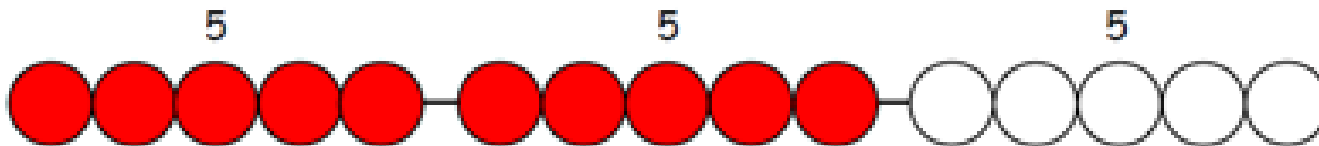
Year 2

Multiply using arrays and repeated addition (using at least 2s, 5s and 10s)

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times

Use practical apparatus:

$$5 \times 3 = 5 + 5 + 5$$



Repeated addition on a number line:

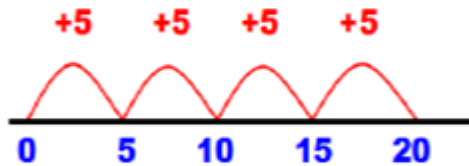
Start from zero

Make equal jumps up on a number line

Write multiplication statements using \times / $=$

$$4 \times 5 = \dots$$

4 lots of 5



$$4 \times 5 = 20$$

Use arrays:

Use arrays to help show the commutative nature of multiplication



$$5 \times 3 = 15$$



$$3 \times 5 = 15$$

Multiplication

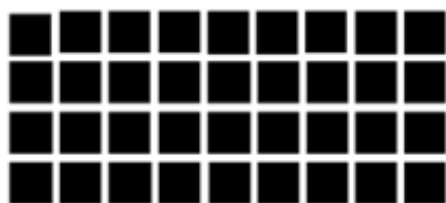
Year 3

Multiply 2-digit numbers by a single digit number

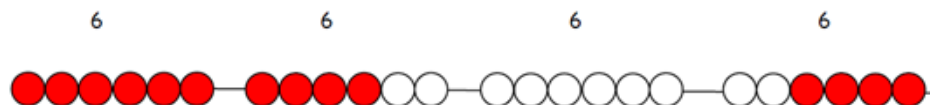
Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times, _times as big as, once, twice, three times, partition, grid method, multiple, product, tens, units, value

Children should be able to: · Partition numbers into tens and units · Multiply multiples of ten by a single digit (e.g. 20×4) using their knowledge of multiplication facts and place value · Recall and work out multiplication facts in the **2, 3, 4, 5, 8 and 10** times tables. · Work out multiplication facts not known by repeated addition or other taught mental strategies (e.g. by commutative law, working out near multiples and adjusting, using doubling etc.) Strategies to support this are repeated addition using a number line, bead bars and arrays.

Children continue to use pictorial representation before progressing onto written form e.g. use of arrays to develop number families. As children become more confident they progress to choosing most appropriate method.



$$9 \times 4 = 36$$



Grid method: Introduce grid method by physically making arrays to represent the calculation.

Eg. $23 \times 8 = 184$

X	20	3
8	160	24

Expanded column method:

Progress towards column method

		2	3	
x			8	
		2	4	
	1	6	0	
	1	8	4	

Multiplication

Year 4 Multiply 2 and 3-digits by a single digit number

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, groups of, sets of, lots of, equal groups, times, multiply, times, as big as, once, twice, three times, partition, grid method, total, multiple, product, sets of, inverse

Develop the grid method:

Eg. $136 \times 5 = 680$

X	100	30	6
5	500	150	30

Encourage column addition to add accurately.

500

150

+30

Develop the expanded column method:

		3	2	7
x				4
<hr/>				
			2	8
			8	0
	1	2	0	0
<hr/>				
	1	3	0	8
<hr/>				
		1		
<hr/>				

Move onto **short multiplication** if and when children are confident and accurate multiplying 2 and 3-digit numbers by a single digit this way, **and** are already confident in “carrying” for written addition.

Children should be able to: •Approximate before they calculate, e.g. 346×9 is approximately $350 \times 10 = 3500$.

Multiply multiples of ten and one hundred by a single-digit, using their multiplication knowledge.

Recall all times tables **up to 12×12**

Multiplication

Next steps

Multiply up to 4-digits by 1 or 2-digits

Multiply decimals with up to 2d.p. by a single digit

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times, partition, grid method, total, multiple, product, inverse, square, factor, integer, decimal, short / long multiplication, carry, tenths, hundredths, decimal

		1	4
x	2	6	
	2	4	
	6	0	
	8	0	
2	0	0	
3	6	4	
1			

Expanded column multiplication for TU x TU:

Short column multiplication for TU x U:

Pupils could be asked to work out a given calculation using the grid, and then compare it to 'your' column method.

$$\begin{array}{r|rr|r} 4 & 300 & 20 & 7 \\ \hline & 1200 & 80 & 28 \end{array}$$

$$\begin{array}{r} 327 \\ \times 4 \\ \hline 1308 \end{array}$$

Introduce long column multiplication for TU x TU

	10	8
10	100	80
3	30	24

$$\begin{array}{r} 18 \\ \times 13 \\ \hline 54 \\ 180 \\ \hline 234 \end{array}$$

$$\begin{array}{r} 3.19 \\ \times 8 \\ \hline 25.52 \end{array}$$

Division

Year 1

Group and share small quantities

Key Vocabulary: share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided into

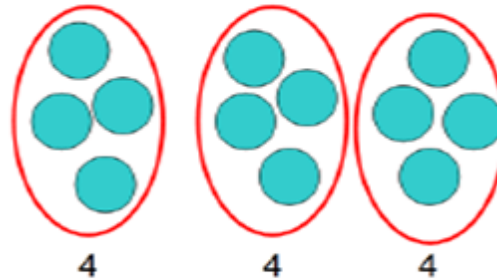
Children use objects, diagrams and pictorial representations to solve problems involving both **grouping** and **sharing** and representing through arrays.

Grouping:



How many groups of 4 can be made with 12 stars = 3

Sharing:



12 shared between 3 is 4

12 balls shared between 3 children = 4

Children should :

- Use lots of practical apparatus, arrays and picture representations.
- Be taught to understand the difference between grouping objects (How many groups of 2 can you make?) and sharing (Share these sweets between 2 people).
- Be able to count in multiples of 2s, 5s and 10s.
- Find **half** of a group of objects by sharing into 2 equal groups.

Division

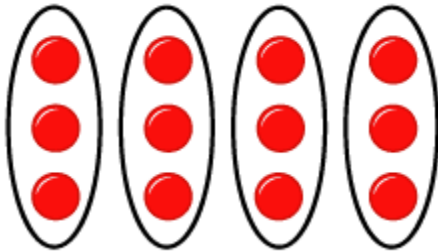
Year 2

Group and share small quantities

Key Vocabulary: share, share equally, one each, two each..., group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over .
Children use objects, arrays, diagrams, pictorial representations and grouping on a number line.

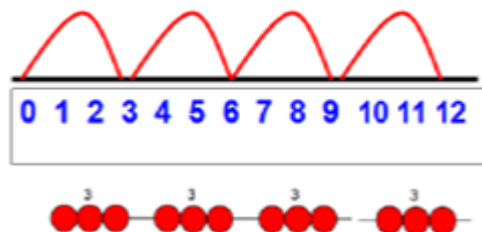
Arrays:

This represents $12 \div 3$, posed as how many groups of 3 are in 12? Pupils should also show that the same array can represent $12 \div 4 = 3$ if grouped horizontally.



$$12 \div 3 = 4$$

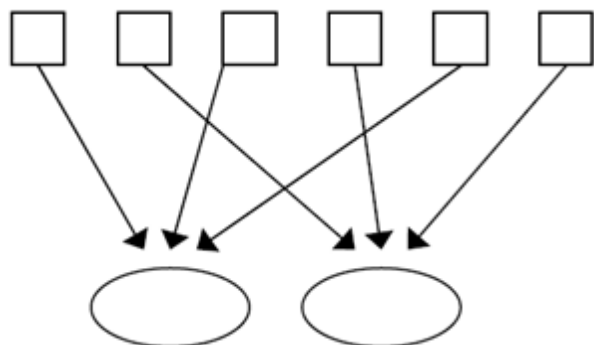
Grouping using a number line: Group from zero in equal jumps of the divisor to find “**How many groups of _ in _ ?**”.



$$12 \div 3 = 4$$

Sharing

6 sweets shared between 2 people, how many do they each get?



There are 6 sweets, how many people can have 2 sweets each?



*This is an important stage in teaching the difference between **grouping** and **sharing**.*

Division

Year 3

Divide 2-digit numbers by a single digit number

Key Vocabulary: share, share equally, one each, two each..., group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple

Grouping using a number line:

Children continue to work out unknown division facts by grouping on a number line from zero. They are taught the concept of remainders. This should be introduced practically and with arrays, as well as being translated to a number line.

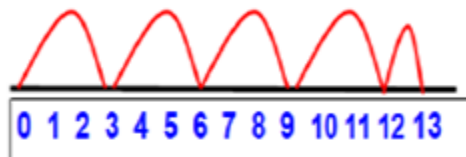
Children continue to use pictorial representation then progress onto written form e.g. use of arrays to develop number families.

Practise times tables as a key to understanding methods. Children should know divisibility rules e.g. multiples of 10 end with a 0.

Children should work towards calculating some basic division facts with remainders mentally for the 2s, 3s, 4s, 5s, 8s and 10s ready for **carrying** remainders across with the short division

$$13 \div 3 =$$

method.



Short division: Once children are secure with division as grouping, **short division** for larger 2-digit numbers should be introduced. Start by introducing the layout of short division by comparing it to an array. **Remind children of correct place value, that 96 is equal to 90 and 6, but in short division, pose:** · How many 3's in 9? = 3, and record it above the **9 tens**. · How many 3's in 6? = 2, and record it above the **6 units**. Initially limit numbers to no remainders or carried numbers. Once children are secure with this method, teach how to use method when remainders occur within the calculation and how to **carry** this remainder to the next digit.

$$\begin{array}{r|rr} & 3 & 2 \\ \hline 3 & 9 & 6 \end{array}$$

$$\begin{array}{r} 18 \\ \hline 4 \overline{) 72} \end{array}$$

Division

Year 4

Divide up to 3-digit numbers by a single digit number

Key Vocabulary: share, share equally, one each, two each..., group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, factor

Develop short division:

Pupils must be secure with the process of short division for dividing 2-digit numbers by a single digit.

They need to understand how to calculate remainders, using this to 'carry' remainders within the

$$\begin{array}{r} 18 \\ 4 \overline{) 72} \end{array}$$

calculation process.

Pupils move onto dividing numbers with up to 3-digits by a single digit, however problems and calculations provided should not result in a final answer with remainder at this stage.

	2	1	8
4)	8	7 ³ 2

When the answer for the first column is zero ($1 \div 5$, as in example), children could initially write a zero above to acknowledge its place, and must always 'carry' the number (1) over to the next

A handwritten long division problem on a blue grid. The divisor 5 is written to the left of the dividend 185. A horizontal line is drawn above the dividend. The quotient 037 is written above the line. The first column shows 1 divided by 5, resulting in 0, with a 1 carried over to the next column. The second column shows 18 divided by 5, resulting in 3, with a 3 carried over to the next column. The third column shows 35 divided by 5, resulting in 7, with no remainder.

$$\begin{array}{r} 037 \\ 5 \overline{) 185} \end{array}$$

digit as a remainder.

Division

Next steps

Divide at least a 4-digit number by a 1 or 2-digit number

Key Vocabulary: share, share equally, one each, two each..., group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, factor, inverse, quotient, prime number, prime factors, composite number (non-prime), common factor **Develop short division (with remainders):**

Children are introduced to examples that give rise to remainder answers, division needs to have a real life problem solving context, where pupils consider the meaning of the remainder depending upon the context of the problem. Include a variety of money and measure contexts for problem solving.

A handwritten short division problem on a blue grid. The divisor 8 is written to the left of the dividend 5309. A horizontal line is drawn under the dividend. The quotient 0663 is written above the line, and the remainder 5 is written to the right of the line, preceded by a small 'r'.

$$\begin{array}{r} 0663 \text{ r } 5 \\ 8 \overline{) 5309} \end{array}$$

The answer to $5309 \div 8$ could be rounded as appropriate to the problem involved.

Children should understand how to express remainders as fractions, decimals, whole number remainders, or rounded numbers.

When calculating a decimal remainder, a decimal point is added after the units because there is still a remainder, and the remainder is carried onto zeros after the decimal point (to show there

was no decimal value in the original number). Keep dividing to an appropriate degree of accuracy

$$\begin{array}{r} 0812.125 \\ 8 \overline{) 6497.000} \end{array}$$

for the problem being solved.